Period 3, Feb 28, 2025

Let u be a differentiable function of x, and let $a > 0$.

1. $\displaystyle\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\frac{u}{a} + C$ **2.** $\displaystyle\int \frac{du}{a^2 + u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$

3. $\displaystyle\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\,\text{arcsec}\,\frac{|u|}{a} + C$

12) $\displaystyle\int x^2(\sqrt{1+5x})\,dx = \int x^2 \cdot \sqrt{u} \cdot \frac{du}{5}$

$u = \boxed{1+5x} \Rightarrow \dfrac{u-1}{5} = x$

$du = 5\,dx$

$\dfrac{du}{5} = dx$

$$\int \left(\frac{u-1}{5}\right)^2 \cdot \sqrt{u} \cdot \frac{du}{5} = \int \frac{u^2 - 2u + 1}{25} \cdot (u^{\frac{1}{2}}) \cdot \frac{1}{5} \cdot du$$

$$\frac{1}{125}\int (u^2 - 2u + 1) \cdot u^{\frac{1}{2}}\,du$$

$$\frac{1}{125}\int (u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + u^{\frac{1}{2}})\,du$$

$$\frac{1}{125}\left[\frac{2}{7}u^{\frac{7}{2}} - 2 \cdot \frac{2}{5} \cdot u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}}\right] + C$$

18) $\displaystyle\int \sec\left(\frac{3x}{2}\right)dx = \int \sec u \cdot \frac{2}{3}\,du = \frac{2}{3}\int \sec u\,du$

$u = \frac{3}{2}x$

$du = \frac{3}{2}dx$

$\frac{2}{3}du = dx$

$\frac{2}{3}\ln|\sec u + \tan u| + C$

$\frac{2}{3}\ln\left|\sec\frac{3}{2}x + \tan\frac{3}{2}x\right| + C$

$\int \sin u\,du = -\cos u + C$

$\int \cos u\,du = \sin u + C$

$\int \tan u\,du = -\ln|\cos u| + C$

$\int \cot u\,du = \ln|\sin u| + C$

$\int \sec u\,du = \ln|\sec u + \tan u| + C$

$\int \csc u\,du = -\ln|\csc u + \cot u| + C$

17) $\int \frac{x+7}{x^2+8x+25}dx = \int \frac{x+7}{u}\cdot\frac{du}{2(x+4)} = \int \frac{(x+4)+3}{u}\cdot\frac{du}{2(x+4)}$

$u = x^2+8x+25$

$du = (2x+8)dx$

$\frac{du}{2x+8} = \frac{du}{2(x+4)} = dx$

$\int \frac{x+4}{x^2+8x+25}dx + \int \frac{3}{x^2+8x+25}dx$

$\int \frac{\cancel{x+4}}{u}\cdot\frac{du}{2\cancel{(x+4)}} + \int \frac{3}{x^2+8x+25}dx$

$\frac{1}{2}\int\frac{du}{u} \quad +$

$a=1$

$b=8$

$\frac{b}{2} = \frac{8}{2} = 4$

$\left(\frac{b}{2}\right)^2 = (4)^2 = 16$

$\underbrace{x^2+8x+16}_{(x+4)^2} \underbrace{-16+25}_{+9}$

$\frac{1}{2}\ln|x^2+8x+25| + \int\frac{3}{(x+4)^2+3^2}dx$

$u = x+4$

$du = dx$

$\frac{1}{2}\ln|x^2+8x+25| + 3\int\frac{du}{u^2+3^2}$

2. $\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$

$\frac{1}{2}\ln|x^2+8x+25| + \cancel{3}\cdot\frac{1}{\cancel{3}}\arctan\frac{u}{3} + C$

$\frac{1}{2}\ln|x^2+8x+25| + \arctan\frac{x+4}{3} + C$

11) $\int x\sqrt{4x+3}\,dx = \int x\sqrt{u}\cdot\frac{du}{4} = \int \frac{(u-3)}{4}\cdot u^{\frac{1}{2}}\cdot\frac{du}{4} = \frac{1}{16}\int (u-3)u^{\frac{1}{2}}\,du$

$u=4x+3 \Rightarrow \frac{u-3}{4}=x$

$du=4dx$

$\frac{du}{4}=dx$

$$\frac{1}{16}\int\left(u^{\frac{3}{2}}-3u^{\frac{1}{2}}\right)du$$

$$\frac{1}{\cancel{16}\,8}\cdot\frac{\cancel{2}}{5}\cdot u^{5/2}-\cancel{3}\cdot\frac{2}{\cancel{3}}\cdot u^{\frac{3}{2}}+C$$

$$\frac{1}{40}(4x+3)^{5/2}-2(4x+3)^{3/2}+C$$

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13) $\int_1^{13}\frac{x}{\sqrt{2x-1}}\,dx = \int_1^{25}\frac{\frac{u+1}{2}}{\frac{\sqrt{u}}{1}}\cdot\frac{du}{2} = \int_1^{25}\frac{u+1}{2}\cdot\frac{1}{u^{\frac{1}{2}}}\cdot\frac{du}{2}$

$u=2x-1 \Rightarrow \frac{u+1}{2}=x$

$du=2dx$

$\frac{du}{2}=dx$

$u=2x-1$

$x=13$

$u=2(13)-1=25$

$u=2(1)-1=1$

$$\frac{1}{4}\int_1^{25}\frac{u+1}{u^{\frac{1}{2}}}\,du = \frac{1}{4}\int_1^{25}\left(\frac{u^1}{u^{\frac{1}{2}}}+\frac{1}{u^{\frac{1}{2}}}\right)du$$

$$\frac{1}{4}\int_1\left(u^{\frac{1}{2}}+u^{-\frac{1}{2}}\right)du$$

$$\frac{1}{4}\left[\frac{2}{3}u^{3/2}+2u^{\frac{1}{2}}\right]_1^{25}$$

$$\frac{1}{4}\left[\frac{2}{3}(25)^{3/2}+2(25)^{\frac{1}{2}}\right]-\frac{1}{4}\left[\frac{2}{3}(1)^{3/2}+2(1)^{\frac{1}{2}}\right]=$$

$$\frac{1}{4}\left[\frac{250}{3}+10\right]-\frac{1}{4}\left[\frac{2}{3}+2\right]$$

Keep going

16) $\int \frac{7}{\sqrt{25-81x^2}}\,dx = \int \frac{7}{\sqrt{(5)^2-(9x)^2}}\,dx$

$u = 9x$
$du = 9\,dx$
$\frac{du}{9} = dx$

$\int \frac{7}{\sqrt{5^2-u^2}} \cdot \frac{du}{9}$

$\frac{7}{9}\int \frac{du}{\sqrt{5^2-u^2}} = \frac{7}{9}\arcsin\frac{u}{5} + C = \frac{7}{9}\arcsin\frac{9x}{5} + C$

1. $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$

3) $\int_1^{\sqrt{2}} x \cdot 2^{-x^2}\,dx = \int_{-1}^{-2} \cancel{x} \cdot 2^u \frac{du}{-2\cancel{x}} = -\frac{1}{2}\int_{-1}^{-2} 2^u\,du = -\frac{1}{2}\cdot\frac{1}{\ln 2}\cdot 2^u + C \Big[_{-1}^{-2}$

$u = -x^2$
$du = -2x\,dx$
$\frac{du}{-2x} = dx$

2) $\int a^u\,du = \left(\frac{1}{\ln a}\right)a^u + C$

$-\frac{1}{2}\cdot\frac{1}{\ln 2}\cdot 2^{-2} - \left[-\frac{1}{2}\cdot\frac{1}{\ln 2}\cdot 2^{-1}\right]$

$-\frac{1}{8\ln 2} + \frac{2\cdot 1}{2\cdot 4\ln 2} = \frac{1}{8\ln 2}$

$Y = 6-2x$ Quad I

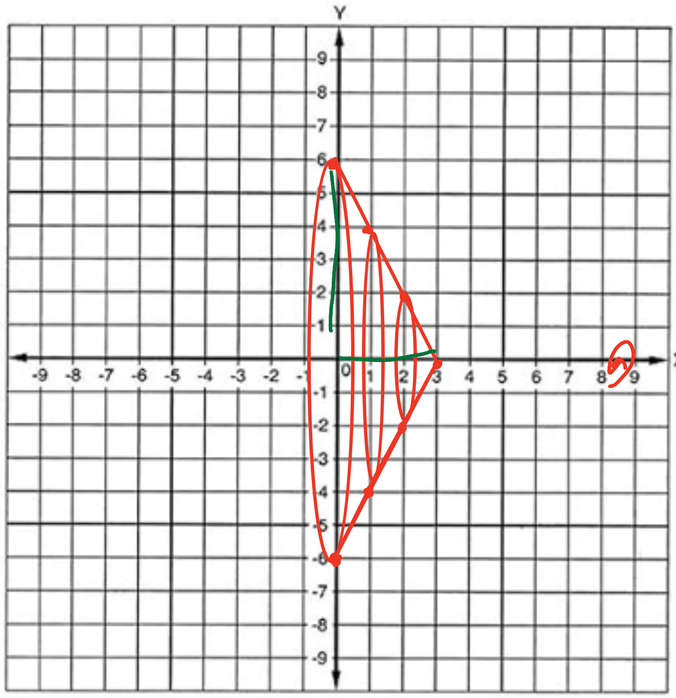
Rotate around x-axis

Find Volume

Volume of cone $= \frac{1}{3}\pi r^2 h$

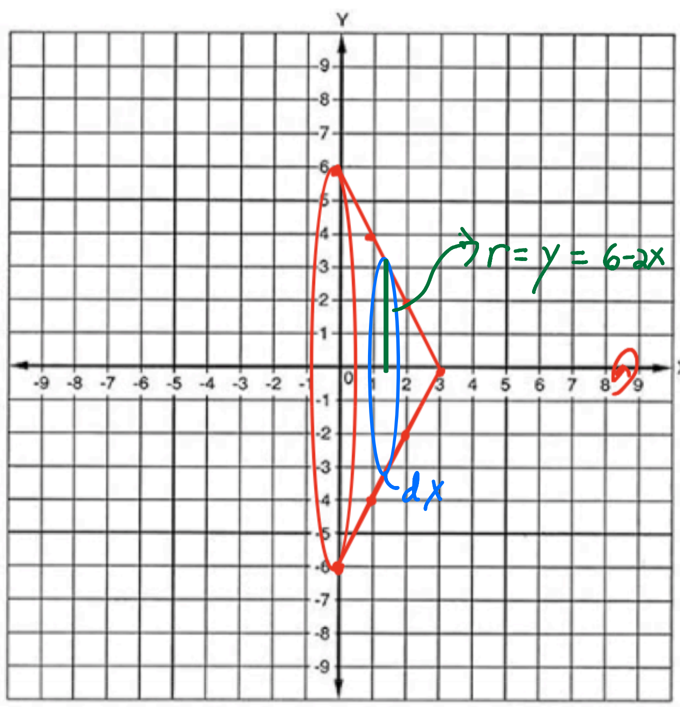
$\left(\frac{1}{3} \cdot \pi \cdot 6^2 \cdot 3\right)$

36π



$Y = 6-2x$

disc method $= \pi r^2 \cdot h$



$$\int_0^3 \pi r^2 \, dx = \int_0^3 \pi y^2 \, dx$$

$$\int_0^3 \pi (6-2x)^2 \, dx$$

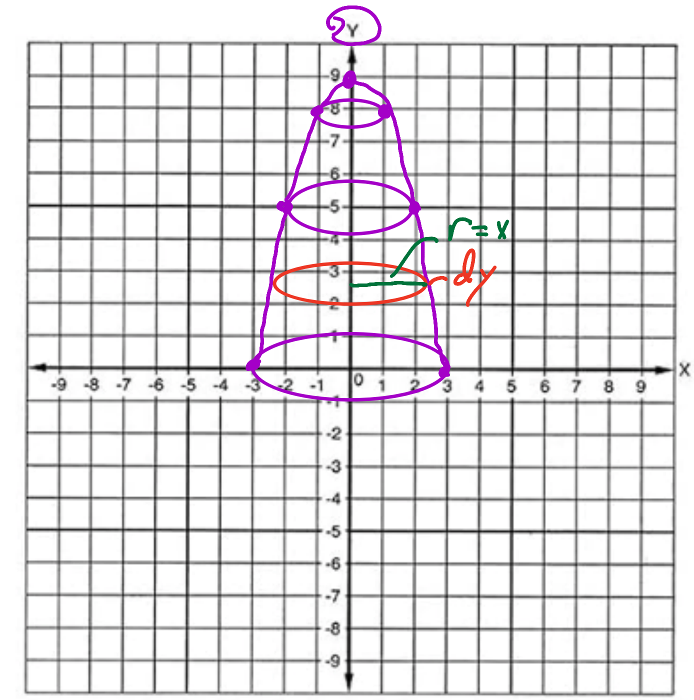
$$\pi \int_0^3 (36 - 24x + 4x^2) \, dx$$

$$\pi \left[36x - 12x^2 + \frac{4}{3}x^3\right] \Big|_0^3$$

$$\pi \left[36(3) - 12 \cdot 9 + \frac{4}{3} \cdot (3)^3\right] - \cancel{\pi[0-0+0]}$$

$$\pi [108 - 108 + 4 \cdot 9] - 0$$

$$36\pi$$



$y = 9 - x^2$ Quad I

around y-axis

$$\int_0^9 \pi r^2\,dy = \int_0^9 \pi x^2\,dy$$

$$\cancel{y} = 9 - \cancel{x^2}$$
$$+x^2 \qquad +x^2$$
$$-y \qquad -y$$
$$x^2 = 9 - y$$

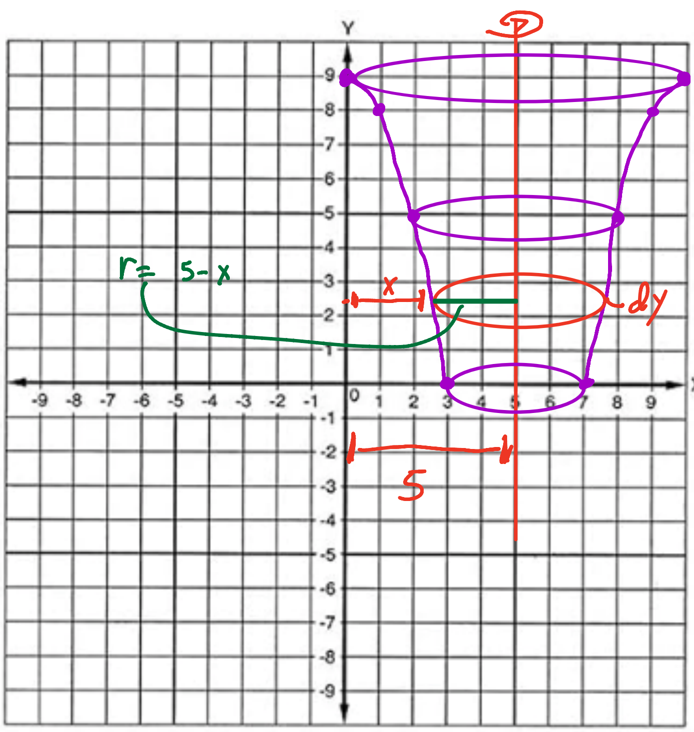
$$= \pi \int_0^9 (9 - y)\,dy$$

$$\pi\left[9y - \tfrac{1}{2}y^2\right]\Big|_0^9$$

$$\pi\left[9\cdot 9 - \tfrac{1}{2}(9)^2\right] - \cancel{\pi\left[9\cdot 0 - \tfrac{1}{2}(0)^2\right]}$$

$$\pi\left[\frac{81\cdot 2}{1\cdot 2} - \frac{81}{2}\right] = \frac{\pi 81}{2}$$

$$\pi\left(\frac{162}{2} - \frac{81}{2}\right) = \frac{8\pi}{2}$$



$y = 9 - x^2 \Rightarrow x = \sqrt{9 - y}$

Rotate around $x = 5$

$$\int_0^9 \pi r^2\,dy$$

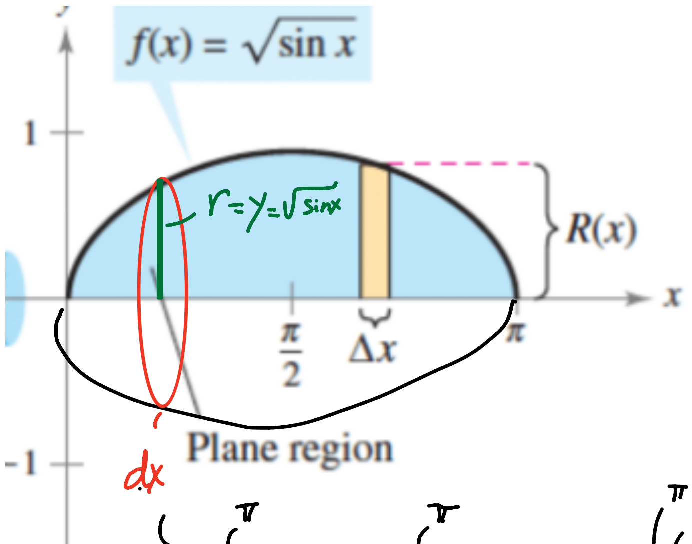
$$\int_0^9 \pi (5 - x)^2\,dy$$

$$\int_0^9 \pi \left(5 - \sqrt{9 - y}\right)^2 dy$$

$$\pi \int_0^9 \left(25 - 10\sqrt{9 - y} + 9 - y\right) dy$$

$$\pi\left[34y - \tfrac{1}{2}y^2\right]\Big|_0^9 + \pi \int_0^9 -10\sqrt{9 - y}\,dy$$

$U = 9 - y$



$$\int_0^{\pi} \pi r^2 dx = \int_0^{\pi} \pi (y^2) dx = \pi \int_0^{\pi} \left(\sqrt{\sin x}\right)^2 dx = \pi \int_0^{\pi} \sin x dx$$

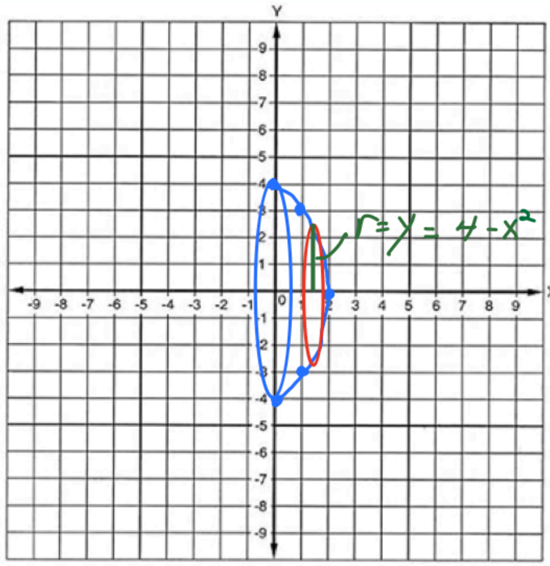
$$\pi\left[-\cos x\right]_0^{\pi}$$

$$-\pi \cos \pi - \left[-\pi \cos 0\right]$$

$$-\pi(-1) + \pi(1)$$

$$\pi + \pi = 2\pi$$

Rotate around x-axis



$$\int_0^2 \pi(4-x^2)^2\,dx$$

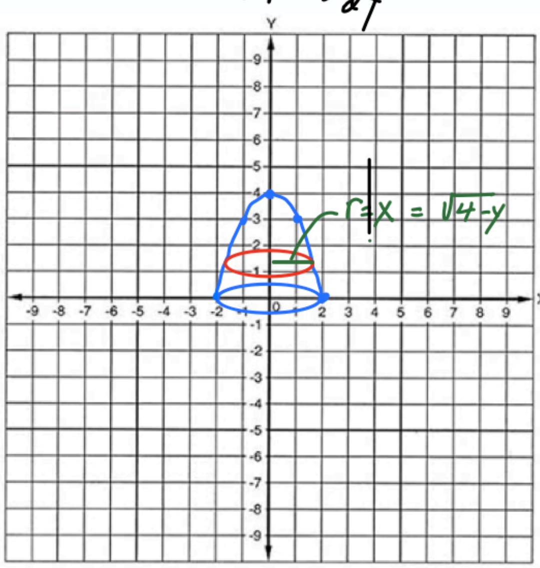
$$\pi\int_0^2 (16-8x^2+x^4)\,dx$$

$$\pi\left(16x-\frac{8}{3}x^3+\frac{1}{5}x^5\right)\Big|_0^2$$

$$\pi\left(16(2)-\frac{8}{3}(2)^3+\frac{1}{5}(2)^5\right)-\pi\left(16(0)-\frac{8}{3}(0)^3+\frac{1}{5}(0)^5\right)$$

$$\pi\left(32-\frac{64}{3}+\frac{32}{5}\right)$$

$\pi r^2 \cdot dy$



$y = 4 - x^2$ in Quad I

x	y
0	4
1	3
2	0

Rotate around

a) the y-axis

Volume $$\int_0^4 \pi(\sqrt{4-y})^2\,dy$$

$$\pi\int_0^4 (4-y)\,dy$$

$$\pi\left[4y-\frac{1}{2}y^2\right]\Big|_0^4$$

$$\pi\left[4\cdot4-\frac{1}{2}(4)^2\right]-\pi\left[4\cdot0-\frac{1}{2}(0)^2\right]$$

$$\pi[16-8]=8\pi$$